IJICSE
Open Access

Contents lists available at www.ijicse.in

International Journal of Innovative Computer Science & Engineering

Journal Index In ICI World of Journals – ICV 2016 – 69.57

Volume 5 Issue 6; November-December 2018; Page No. 01-08

Big Bang: Lemaitre's model possesses the mathematical characteristics else

Dr. Shobha Lal

Professor of Mathematics and Computing & Dean Faculty of Engineering and Technology,

Jayoti Vidyapeeth Women's University, Jaipur, India

ABSTRACT

This Universe of de Sitter was expanding in an infinite flat space (k = o) but he visualized an empty Universe (ρ=o). the expanding model of the Universe appeared strange to the physicists at that time, about a decade before Hubble's discovery of receding galaxies. In de Sitter's Universe, ρ = p = k= o, when Eqn. yields

$$R(t) = R_o \exp\left[\tfrac{1}{3}c^2 \wedge\right]^{1/2} t$$

Since R/R = H, the Hubble constant, the age of the de Sitter Universe can expressed by

$$\frac{1}{H} = \left(\frac{3}{c^2 \wedge}\right)^{1/2}$$

It can be noticed that if in such an empty Universe we assume further that ∧= o, then we get a static Universe of undetermined extent. The basic weakness of de Sitter Universe is that it is empty and thus unphysics

Keywords: Big Bang,Model, Relativity, Astrophysics, Astronomy, Hyper Geometric, Binary.

Introduction

THE CURVATURE PARAMETER

Albert Einstein himself was the first to use his theory of General Relativity in building a World model in 1917 which, of course, due to several of its basic weaknesses has now reduced to one of historical importance only. At that time the expansion of the Universe was not discovered which came actually after a decade from then. So Einstein adopted the cosmological principle and starting with his relativistic equations he calculated the average density of the Universe. The Einstein universe is spherical with the curvature parameter k = 1 and having a constant radius of curvature, R. so space is unbounded but finite in such a Universe, since R is finite. Equations therefore yield for Einstein Universe (k = 1) and R = constant)

$$\wedge = \frac{1}{c^2R^2} + \frac{8\pi Gp}{c^4}$$

And

$$\rho = \frac{2}{c^2R^2K} - \frac{P}{c^2}$$

Where we have to deduce Eq. and written $K = \frac{8\pi Gp}{c^2}$ whose numerical value is 1.86 x 10^{-27} cm gm^{-1}. The cosmological constant ∧ actually corresponds to a tension which was introduced by Einstein on an ad hoc consideration. The effect of this term is not perceptible over short range of distances as those within the solar system but becomes more and more important with increasing distances so as to be capable of bringing the expansion of the Universe to a halt at a certain stage. This last statement, of course, is not relevant to the Einstein Universe which does not expand at all Einstein considered ∧ as producing an effective repulsion against the gravitational attraction.

Two basic defects of the Einstein universe are that he supposed the Universe to be static (not variablewith time) and completely devoid of radiation with a temperature of 0 Kelvin, Subsequent observations have proved that both of these assumptions are wrong.

Corresponding author: Dr. Shobha Lal

About the same time in 1917, the Dutch astronomer W. desitter proposed another model of the universe which also was based on the cosmological principle and the relativistic equations. This Universe of de Sitter was expanding in an infinite flat space (k = o) but he visualized an empty Universe (ρ=o). the expanding model of the Universe appeared strange to the physicists at that time, about a decade before Hubble's discovery of receding galaxies. In de Sitter's Universe, ρ = p = k= o, when Eqn. yields

$$R(t) = R_o \exp [\frac{1}{3}c^2 \wedge]^{1/2} t$$

Since R/R = H, the Hubble constant, the age of the de Sitter Universe can expressed by

$$\frac{1}{H} = (\frac{3}{c^2 \wedge})^{1/2}$$

It can be noticed that if in such an empty Universe we assume further that $\wedge$= o, then we get astatic Universe of undetermined extent. The basic weakness of de Sitter Universe is that it is empty and thus un physics

CONTRACTION

Around 1930. E.E. Lemaitre and A.s. Eddington discovered independently that the Einstein Universe was unstable against small perturbations. This is because the Einstein Universe is pictured as being held in equilibrium by equal and opposite forces of gravitational attraction and cosmological repulsion (incorporated by $\wedge$). This precarious state of equilibrium, if slightly disturbed, will give rese to instability. The universe, if disturbed by a slight expansion initially, will subsequently continue to expand with ever increasing speed. If, on the other hand, the intial disturbance is produced by slight contraction, the Universe will subsequently, continue to contract with ever increasing speed. The Lemaitre-Eddington model of the Universe therefore starts with Einstein's static state and undergoes through the first alternative after getting some initial disturbance: that is, it continues to expand subsequently with ever increasing speed this Universe, therefore, has an infinite past spentin Einstein state and starting from some finite epoch in the past will continue to expand for an infinite future. The galaxies and stars were formed over a finite period during the initial expanding phase. The works of some authors such as Lemaitre. Mc Creea and Mc Vittie have shown that the static Einstein Universe would expand after the condensation fo galaxies started for some reason.

The Lemaitre-Eddington model which was well accepted for sometime, however, soon became the subject of various criticism: particularly, when the formation of elements began to be considered seriously. With a view to get rid of these weaknesses, Lemaitre proposed another model of the Universe also based on the theory of General Relativity. In this model, Lemaitre suggested tha the entire matter of the Universe was originally in a single chunk of extremely high density and temperature but occupying a small volume. This body was called by Lemaitre the primeval Atom. Due to some reason this superdense body became unstable, a gigantic explosion took place and material started flowing in all directions at very high speeds. The elements were formed during the early period of rapid expansion. The Universe was like Einstein universe when the material condensations took place to from galaxies. These condensations triggered the second phase of expansion which has since been continuing with ever increasing velocity as depicted.

The model proposed by Lemaitre subsequently established the concept of the Big Bang which has since been variously discussed by authors. It envisages the evolutionary cosmology. According to this model the Universe started from the explosion of the Primeval Atom at some particular epoch in the past, the galaxies were formed during a particular time period, which therefore have since been aging and elolving together. All the various models constructed on the basis of the various versions of the Big Bang theory are therefore called the evolutionary models of the Universe. The age of the Universe discussed is obtained on the basis of this theory.

Lemaitre's model possesses the mathematical characteristics that k> 0, $\wedge > \wedge_c$, a critical value of $\wedge$ defined by

$$\wedge = \frac{k^3}{16\pi^2 G^2 \rho^2 ((t_o)}$$

And r > o, but gradually decreasing as the expansion proceeds.

The friedmann Models

A variety of cosmological models were constructed by the Russian mathematician A. Friedmann in early 1920s. friedmann used the Einstrein relativistic equations with cosmological principle and simplified those equations further by taking $\wedge$ = o. all the cosmological constant enumerated above so far were constructed with non-zero values of the cosmological constant $\wedge$- which is essentially equivalent toa tension against which work has to be done if expansion is to continue. The introduction of $\wedge$ by Einstein in his equations in an arbitrary fashion became all along a subject of controversy among cosmologists. Friedmann simplified the matter just by taking $\wedge$ = o. this was indeed a bold step which greatly simplified the mathematical formalism.

Friedmann has considerd models of both closed and open Universe having one characteristic common to all of them, viz., they all start at some time t = o in an extremely dense state and subsequently evolve in time t in curved space defined by either k = + 1 (spherical) of k= -1 (hyperbolic). In the initial dense state both pressure and density are high which subsequently continue to decrease as the, expansion proceeds. The models with k = +1 (and q> ½) start from dense state, expand to some maximum value of the scale factor R(t), then start contracting again to reach ultimately the initial dense state. Such cycle can be repeated indefinitely giving an oscillating Universe. In such a special case the universe might continue from an indefinite past to an indefinite future, the scale factor R(t) following a sequence of hoops in time t as shown in Fig. 21.6 Both a zero and a non-zero finite value of the scale factor at maximum contraction can be visualized. In the former case, the density of the scale factor at maximum contraction can be visualized. In the former case, the density becomes infinite, while it is finite in the latter case.

Other Friemann models with $\wedge$ = o, k= -1 (o ≤ q ½) start from dense states and continue to expand indefinitely in the curved hyperbolic infinite space.

All the models described above are relativistic models of the universe in curved space constructed on the basis of Einstein's equations of General Relativity with various values of $\wedge$ and k. An entirely new model of the expanding Universe was proposed independently by H. Bondi and T. Gold and by Hoyle. This Universe is in flat space (k = o) and is characterized by the decelerating parameter value q = 1: that is, the Unversed expands with an acceleration. All the relativistic models described so far postulate the cosmological principle and the conservation of the mass-energy. The steady state model, on the other hand, goes much beyond in postulating the, perfect cosmological principle and discarding the most vital physical law of the conservation of mass-energy in the Universe. The validity of the perfect cosmological principle requires that the universe is isotropic and homogeneous at all time. Since the universe is expanding, the principle demands that new matter must be created to maintain a constant density of the universe. The newly created matter must replenish that which vanish beyond the limit of the observable Universe. The most remarkable feature of the theory is that the new matter (believed tube hydrogen atom) is supposed to be created out of nothing in a creation field called the C-field, Matter, therefore, requires to be continuously created in the Universe according to this theory, It turns out, however, that the rate of creation of new matter to replenish the lost amount is very low. The rate of volume expansion of a sphere of radius R(t) is given by

$$\frac{d}{dt}\left[\frac{4}{3}\pi R^2 (t)\right] = 4\pi R^2(t)\,\frac{dR(t)}{dt} = 4\pi R^2(t)H(t)$$

By using Hubble's law. If the density is to be maintained at some constant value ρ, then the rate of matter creation within the sphere of radius R(t) becomes $4\pi R^2(t)H(t)\rho$. Therefore, the rate of matter creation per unit volume is$3H(t)\pi$. This is numerically – 10^{-47} gm c^{-3} s^{-1} or about one H atom per cubic kilometer of space every five years. This appears to be such a small rate creation that it cannot be observationally verified and "it probably canbe done by a lesser Being than the Almighty God". But the space is so vast that even this small rate sill sum up, when taken over the entire space, to the creation of more than a thousand stars every second. The question therefore naturally arises: where does this enormous amount of energy come from? We do not have any satisfactory answer.

The continuous creation theory therefore predicts that in order to satisfy the perfect cosmological principle, new galaxies must be condensed out of the newly created matter where new stars will be formed. These galaxies, taking part in the expansion of the Universe, will separate to greater and greater distances from one another while, at the same time, will age, grow old and eventually proceed toward the end of their vies. Consequently, in any given volume of space there must always be found the same proportion of old and young galaxies formed at different ages. The universe, acceding to this theory, has neither a beginning nor an ended either in space Orin time. It is infinitely large and infinitely old, having an infinite future.

Although faced by many difficulties in view of the current observational status of the universe, the steady-state theory has evoked great interest among the cosmologists since the time of its formulation. since the relativistic models also are found to be vulnerable against observational test, the steady-state theory was considered for some time as an alternative, and a very good one at that. The strongest argument against this theory for which many physicists are skeptical about the soundness of it is most interesting feature, the continouos creation of matte, violates the law of conservation of energy, which is regarded by the physicists as one of the most sacrosanct laws of physics. But the propounders of the steady state theory attempt to counteract the objection by arguing that matter in the Universe has been created any way at some phase of the Universe. It is no more difficult to conceive the continuous creation of matter than that itt was created as the primordial atom at some particular time. Moreover, the concept of the continuous creation and of the infinite extension in both space and time of the steady sate universe has imparted it a great philosophical charm. That is why, in spite of its weaknesses against several observational aspects, the theory has been adhered to by many cosmologists with the introduction of certain modifications from time to time.

SCALAR FIELD

This theory of the model of theUniverse was proposed by C. Brans and R.H. Dicke on the basis of a modification of the relativistic equations of Einstein. As we have already seen, the tensor g_{ij} in Einstein equations of General Relativity given by Eq. (21.28) represents the distribution of the gravitational field. To this tensor field Brans and Dicke have added a small scalar field thereby modifying the relativistic equations. Although the added scalar field is small, its effect becomesfar-reaching when considered against the theoretical evolution of the cosmological models. But the observationl tests of its validity are quite difficult as very great accuracy of measurements is necessary to observe its small difference from the results obtained on the basis of Einstein's theory. In one case, however, viz.., the motion ofterrestrial planets, where Einstein'stheory has been found to be correct, the Grans-Dicke theory has been found to disagree. Nevertheless, whenthe cosmologists in their hectic search for a most appropriate theory that will correctly predict all observable aspects of the universe have been eluded so far, the Brans-Dicke theory must be regarded as a valuable contribution in the race. Whether it will stand the test of time againstthe background of observational verification, we donot know. But its importance lies in the fact that it has initiated new ideas in cosmology and has encouraged to adopt new lines of observational tests.

DEPICTED UNIVERSE

We have discussed in the last section some of the important World models that have been proposed by cosmologists maintaining different viewpoints. These models are principally of two different types, barring the static model of Einstein and the empty model of de Sitter which now have been reduced to of historical interest only. The two competitive types are the evolutionary cosmology and the steady-state cosmology. According to the former, the Universe started at a particular epoch some (1-2) x 10^{10} years ago with a Big Bang from an extremely dense and got state. Whether it started from a singularity or finite size and whether it will expand indefinitely or fall back again to the original dense and hot sate, still remain a controversy. Some authors have tried to show that the universe might have started from an initial size of the Earth's orbit at a temperature of 10^{12} K. The heavy elements were formed shortly after the time of explosion. The galaxies and stars were all formed at subsequent epochs. They are all aging and evolving since then, i.e. the Universe as a whole is evolving in time. This is in fact, the picture of the Universe depicted, in general, by all evolutionary models of cosmology. These models are all based on the theory of General Relativity. According to the Steady-State Cosmology, on the other hand, the Universe is infinite both in time and extent and the expansion rate of the Universe must increase in time. Any finite volume of the inverse must contain a homogeneous mixture of galaxies of all different ages so that no question of evolution is relevant in this case.

Among such a variety foodless, if they are really exhaustive, only one should emerge as true when subjected to observational scrutiny. Unfortunately, very distant galaxies require to be observed for a decisive test, but at such distances several difficulties arise in interpretation of observed properties. One very important test is yielded by the observation of average mass density of the universe and the corresponding deceleration parameter. This will give a clue to the understanding as to whether the Universe is expanding at a constant rate or the rate of expansion is slowing down. Unfortunately, such a discrimination cannot be made with galaxies for which $Z < 0.2$. for these nearby galaxies the velocity-distance curve is straight line . The interpretation of such diagrams rests on two basic assumptions . First, the measured red shifts of galaxies are Doppler shifts (also called "Cosmological" red shifts), implying that by measuring shifts of spectral lines we actually measure the velocities of the sources. Sex only, the brightest members in clusters of galaxies have the same absolute luminosity so that by measuring different apparent magnitudes of such members we actually measure their different distances. With these assumptions has been drawn to represent the redshift-magnitude diagram (which is equivalent to the velocity-distance diagram withabove two assumptions) for the brightest members of 38 clusters of galaxies. The relation is fairly well represented by a straight line, meaning thereby, that the expansion of the Universe is uniform ($q < o$). If q is non-zero, the galaxies will be either decelerate ($q>0$) of accelerate ($q< o$) and in either case the red shift-magnitude relation will be non-linear. But this nonlinearity is revealed only at very great distances . It is to be noted from the figure that all the four curves corresponding to q= +2, +1, 0 and-1 are almost linear and coincident until distances corresponding to about Z – 0.2 are reached. The curves separate at greater distances and in order to discriminate between the various world models, one has to observe galaxies at these distances. If the distant galaxies are found to lie along the curve marked 0, then an uniformly expanding model of the universe will be a correct picture. If the galaxies are found to lie along some curve lying intermediate between those marked 0 and +1 then the expansion of the Universe is slowing down. But in this case the Universe will never stop although its rate of expansion will gradually decrease. Both open and flat models of the Universe are relevant to this case. A peculiar phenomenon will, however, occur if the galaxies are found to lie along the curse marked+1 or above. In this case the Universe will expand to maximum extension and then retrace back its path in contraction. Calculations reveal that the universe contracts to a state of high temperature and high density and at the end of contraction it will vanish to a singularity. Some authors however, have suggested that the rotation of the Universe as a whole may spare it from going back to a singularity. The universe in the case will bounce back form a dense state of finite size which subsequently leads it to an unending series of oscillations as depicted .

If, on the other hand, the distant galaxies fall along a curve lying between those represented by 0 and -1, then the Universe is expanding with an increasing rate and will extend to infinity. In particular, if the galaxies are found to lie along the curve q = -1, the Universe is in steady-state.

As shown in Fig. 21.7, the present observational status would suggest that the distant galaxies rather lie along the curve for +1. In particular, the correspondence between the galaxies and the curse marked -1 appears poor. A comparatively batter fit is suggested between the galaxies and a curve lying anywhere between those marked 0 and +1 which means, that we at present are living in an expanding but decelerating Universe. It appears quite different from a steady-state universe. But a careful consideration of the entire problem would rather suggest that we will be mistaken to draw any definite conclusion on the basis of the current observational data. Some inherent uncertainty may spoil the entire basis of our inference. The plausible uncertainty may arise from our second basic assumption enumerated above. Galaxies at great distances may not be basically of the same intrinsic luminosity as those close to us. In fact, when we are observing galaxies several billion light years away, we are actually observing them as they were several billion years ago. It may be unlikely that a galaxy will remain essentially unchanged for such a long period. Since the galaxies evolve in time and their stellar content changes with evolution, it seems likely that their intrinsic luminosity also changes with the change of the stellar content. This is perhaps more appropriate for the largest galaxies which are observed for the purpose. Thus the assumption that the largest galaxies in clusters at different distances are essentially of the same intrinsic luminosity, is likely to introduce uncertainty when greater depths of space are concerned. An uncertainty in luminosity even by a factor of two may disturb the entire basis of our conclusion, because galaxies may then fit with a curve lying anywhere between those marked by -1 and +1. In the absence of more accurate observation of distant galaxies, it will be therefore, unwise to draw any firm conclusion regarding the correct model of the Universe.

However, some observational facts have been gathered which most cosmologists believe today, supply evidences against a steady-state Universe. In fact, these observations are believed to favor a Big Bang. But again, in view of the great complexity of things and ingerent uncertainties lying in measurements as well as in interpretations, the reader may be warned to be too optimistic in drawing an unambiguous conclusion.

The first observational fact that speaks in favor of a Big Bang (and against a steady-state universe) is the detection of the so-called 3K (more correctly, 2.7K) isotropic background radiation. It was suggested by Gamow and by Dicke that if the present expansion should be detectable even now. This radiation which was extremely intense and of very high energy during explosion was mostly absorbed when initially flowed through the dense, hot and opaque matter. But as the matter thinned out due to subsequent rapid expansion, some of the radiation escaped encounter with matter and was thus spared. Under these conditions, we should be able to detect a remnant of this radiation that may flow to the Earth. Both the above authors predicted, however, that the original high energy radiation representative of a very hot body, while coming from very far away in space will be very far away in space will be very greatly reshifted. As a result, the original radiation in X-rays is likely to be detected in radio waves as if radiated from a cold body at a temperature of a few degrees Kelvin.

This remarkable theoretical prediction was actually verified when A.A. Penzias and R.W. Wilson of Bell Telephone Laboratories detected in 1965 an isotropic flux of background radio radiation at 7.35 cm. the isotropy of this radiation was so perfect that the only plausible explanation required was to assign it to extraterrestrial origin. More detailed and sensitive observations at several more wavelengths not only confirmed the correctness of this assumption but also showed that the measured radiation at these wavelength correspond to that of a blackbody at about 3K. an additional co confirmation of the existence of this isotropic radiation comes from the observation of lines of CN in spectra of the stars ρ Oph and ρ Per. The lines suggest that the level of excitation of CN molecules is the same as it would be if they were bathed in radiation corresponding to a wavelength of 306 cm which again is characteristic of a blackbody at temperatures around 3.7K. this remarkable coincidence between the theoretical prediction and observed results has encoumany cosmologists to believe that in this 3 K radiation, we are actually seeing the Big Bang of the extremely dense and hot primordial atom of Lemaitre.

The second observational fact apparently speaks more against the steady state universe than it votes in favor of the Big Bang. This comes from a study of the distribution of Quasars in space. It is believed that Quasars are objects passing through a particular phase of their evolution. According to the steady-state theory such objects should be uniformly distributed in space, because every volume of space must contain a homogeneous mixture of object of all ages. But the study of the distribution of a homogeneous sample of Quasars reveals that a large majority of them lie very far away, from us. This implies that the Quasars are objects that mostly existed in early stage of the Universe and are infant reminiscent of the early evolutionary phase of the Universe, the like of which we rarely see to day. If this interpretation is correct then it certainly violates the concept of the perfect Cosmological Principle which is pivotal to the steady-state cosmology.

We conclude this chapter with a note of warning: The subject of cosmology has been a meeting place of contradictory (of at least alternative) theories and observations. We are not even sure whether the observed redshifts are cosmological. We just assume it in the absence of a better alternative. But many physicists have questioned its validity. We do not know yet the true physical nature of the Quasars on the basis of which the soundness of the steady-state cosmology has been questioned. Many authors have suggested that new galaxies do from. The observation of peculiar features in galaxies has confirmed this idea. Groups of galaxies strung along a line and tubular connections between galaxies definitely suggest that they cannot so remain for a very long time, and therefore, must have been of rather recent origin. Such observations therefore lend support to the continuous creation hypothesis. Thus the present observational status does not allow any cosmological model to be either wholly accepted or wholly rejectd and we have to keep our choice open for some more time to come.

POLARIZATION STATE.

The cosmic microwave background radiation was predicted in 1948 by George Gamow, Ralph Alpher and Robert Herman. It was first observed by Arno Penxias and Robert Wilson in 1965 at the Bell Telephone Laboratories in Murray Hill, New Jersey during the calibration of the horn radio antenna devised to track the satellite echo. They found that the noise was independent of the direction of antenna and this indicated that the noise was of cosmic origin. Subsequent studies showed the radiation to have a temperature of 2.7K and the spectrum was a thermal blackbody curve. The black body nature of the spectrum, indirectly supports, the big bang theory of the formation of the Universe. One of the profound observations of the 20th century has been that the universe is expanding. This expansion implies a smaller, denser and hotter universe in the distant past. At this high density matter and radiation were in thermal equilibrium. As the universe expanded, both matter and radiation cooled and at 3000 K, elections joined the atoms breaking the thermal contact and thus matter decoupled from radiation making the universe transparent from an opaque state. Before the ‘decoupling era’ cosmic microwave background photons easily scattered off electrons. This process of multiple scattering produces what is called a ‘thermal’ of black body (BB) spectrum of photons. So according to the big bang theory, there should have been a BB spectrum and this was indeed measured with FIRAS (Far Infrared Absolute Spectrophotometer) experiment on NASA’S (National Aeronautics and Space Administration) COBE (Cosmic Background Explorer) (Fig. 21.8) satellite. To a first approximation we expect the photos in the universe to have a BB spectrum.

Now there is a question. If the universe was so uniform, then how were the different structures in it formed that we see today? There must have been some bumps in the early universe that grew to create the structures that we see today. In 1922, COBE detected the bups (ΔT_b) for the first time

Let I_v be the specific intensity of light (incident energy per unit area, per unit solid angle, per unit frequency, per unit time). Then,

$$I_v = \frac{2hv^3}{c^2} n_v$$

Where v is the frequency, n_v (v) is the quantum mechanical occupation number, i.e. the number of photons (in each polarization state) per unit phase space volume measured in units of h^3, h is plank’s constant. It is assumed that the light is not linearly polarized so that there are an equal number of photons in each polarization state. Therefore, a black body (BB) spectrum

$$n_v = \frac{1}{\exp\left(\frac{hv}{kT}\right)-1}$$

where T is temperature. If there is a small deviation from BB spectrum, and the temperature measured is $T_{v'}$ which is a good fit to $T_{b'}$ then, the fluctuation $\Delta T_b = T_b - T_v$ is the differentiation of Eq. (21.43) w.r.t. v and is given by,

$$\Delta T_b = T_b - T_{v'} = \frac{(e^x - 1)^2}{x^2 e^x} \quad \frac{c^2 \Delta T_v}{K_2 V^2} \quad \text{where}$$

$$X = \frac{hv}{kT_v}$$

There are three processes which are significant for thermal zing the BB spectrum: (i) Compton scattering (ii) double Compton scattering and (iii) free-free scattering. During the epoch, Compton scattering is efficient. So, if there is a perturbation, the spectrum approaches a Bose Einstein distribution (NBE) via Compton scattering instead of black body and then

$$N^{BE} = \frac{1}{\exp\left(\frac{hv}{kT_v + \mu}\right)-1}$$

Where T_v and μ are are the measured temperature and dimensionless chemical potential. So if initially, there is a thermal distribution of photons at temperature T_v and there is an increase of fraction energy density, without significantly increasing the number of photons, then

$$T_b - T_\gamma \ [1 - \mu\,(0.456 – x^{-1})],\ \Delta U/U = 0.71\mu$$

If, [**μ**] $<9 \times 10^{-5}$ (comparing FIRAS data), then

$$\Delta U/U < 6 \times 10^{-5},\ 10^{-5} < Z < 2 \times 10^{6}$$

Standard Cosmological Model for Anisotropy

Einstein's equation for the evolution of the Universe is,

$$R^2\,(t) = (8/3)\,\pi G \rho R^{-1}(t) + (\wedge/3)R^2 – kc^2$$

Where R describes the size of the universe, G is the gravitational constant, ρ is the present density of the universe, k is a measure of the curvature of space and ∧ is the cosmological constant which can be considered zero energy of a vacuum. If the cosmological term dominates then

$$R^2(t)\ (\wedge/3)R^2 \text{ leading to } R(t) \propto \exp\,\{((\wedge/3)^{1/2}t$$

Therefore, the inflationary theory describes the exponential expansion of space which occurred in the very early Universe. Amplification of initial quantum irregularities then resulted in a spectrum of long wavelength perturbations on scales initially bigger thatn the horizon size. There is a reasonable agreement that the form of fluctuation spectrum coming out of inflation is

$$|\delta_k|^2 \propto k_n$$

Where k is the commoving wave number and n is the 'tilt' of the primary spectrum. The latter is predicted to lie close to 1 (Harrison Zeldovich or 'scale invariant' spectrum). The nature of oscillation in the subsequent stage is acoustic in nature. In the photon baryon plasma dominated era, the pressure of photons tends to erase anisotropies, whereas gravitational attraction of the baryons, which are moving at speeds much less than the speed of light makes them tend to collapse to form dense haloes. These two effects complete to create acoustic oscillations which give the CMBR its characteristics peak structure . the peaks (Doppler peaks) correspond to resonances in which the photons decouple when a particular mode is at its peak amplitude. Also, diffusion damping (Silk damping) contribute to the

Power spectrum for standard CDM. Parameters assumed are $\Omega – 1$, H_0 - 50 kms^{-1} and a baryon fraction of Ω_b - 0.04 suppression of anisotropies on small scales when the treatment of the primordial plasma as a fluid begins to break down. So, on large angular scalars (-2^0), CMB spectrum refects inflation; on intermediate angular scales (-1^0) there are series of Dopper peaks and on smaller angular scales (-1^0) there is sharp decline in the amplitude. Incorporating all these phenomena a model, called standard model, has beenconstructed involving inflation together with cold dark matters (CDM). The power spectrum for, W = 1, H_0 = 50 km s^{-1}

The quantities plotted are $(l^2C_l/2\pi)$ x 10^{-10} vs l, where

$C_I = <| a_{lm}|$, $\Delta T (\theta, \varphi)/T = \sum a_{lm} \ Y_{lm}(\theta, \varphi)$

So, $l (l + 1) C_l$ approximately equals the power per unit logarithmic interval in l. $\theta = 2/1$ where θ is the angular scale. The peaks contain interesting physical signatures, e.g. the first peak at $l = 200$ and $\theta - 1^0$ is the acoustic peak or Doppler' or 'Sakharov' peak. Now, $l_{peak} \propto \Omega^{1/2}$. So, the accurate observed position of the peak gives a rough estimate of the total density of the Universe. Also, height of the peak – Ω_bH_0. From nucleosynthesis, there is constraint on Ω_bH_0. So using Ω_bH_0 and Ω_bH_0 value of H_0 can be determined.

References:

1. Bondi, H. Cosmology, Cambridhe University Press, Cambridge. 1961
2. Bonnor, William, The Mystery of the Expanding Universe. The Macmillan Company, New York, 1963.
3. Contopoulos, D. andKatskis, D. Cosmology (Translated from Greek by M. Petrou and P.L. Palmer), Springer-Veriag, Berlin,1987.
4. Harrison, Edward, R. Cosmology, Cambridge University Press, Cambridge, 1981.
5. Maeder, A. Martinet, L. and Tammann, G. (Eds),observational Cosmology, Geneva Observatory, Switzerland, 1978.
6. Narlikar, J., Introduction to Cosmology, Cambridge University Press, Cambridge, 1993
7. Partridge, R.B.,3K: The Cosmic Microwave Background Radiation, Cambridge University Press, Cambridge, 1995.
8. Peebles, P.J.E. Physical Cosmology, Princeton University Press, Princeton, New Jersey, 1971.
9. Roy Choudhury, A.K., Theoretical Cosmology, Oxford University Press,Oxford, 1979.
10. Roy Choudhury, Springer-Veriag, Berlin, 1992.
11. Sanchez, F., Collados, M. and Rebelo,R.(Eds.), Observational and physical Cosmology, Cambridge University press, Cambridge, 1992.
12. Sciama, D.W.,Modern Cosmology, Cambridge university press, Cambridge, 1971.