ISSN: 2393-8528

Contents lists available at www.ijicse.in

International Journal of Innovative Computer Science & Engineering

Journal Index In ICI World of Journals – ICV 2016 – 69.57

Volume 4 Issue 6; November-December 2017; Page No. 22-29

Radiation from Falling Particle

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ABSTRACT

It has been said earlier that of all particles with a fixed rest mass and a fixed angular momentum and which are not trapped by the black hole, the orbit which minimizes the energy in the stable circular orbit. In case the falling particle has energy greater than the minimum energy required to settle down to a circular orbit and if it is not first trapped by the black hole, it is possible that a dissipative process such as gravitational radiation will cause the test particle to relax to such a minimum energy. In case, it be not possible for the test particle to relax to such a minimum energy, the particle will spiral down the hole and get accreted to it. In such a type of motion, the capture of the test particle is immediate and the effective potential does not attain a minimum value. That there is a direct stable circular orbit at r = (3 +√5)m/2 where the energy of the test particle is given by E/ □~– 0.86 and the last direct stable circular orbit is given by r = m where the energy of the test particle is given by E/ □– 0.58. If in case, the test particle in the orbit r = (3 +√5)m/2, which lies just outside the ergo sphere, begins to loose the energy, it will fall down through the ergo sphere and if it does not get entrapped by the hole, it will settle down in the circular orbit r = m. The particle will follow a spiraling path in falling from the first orbit to the second orbit. Thus, it would be quite interesting to study such an orbit of an accreting particle.

Keywords: Radiation, Energy, Value, Angular Velocity, Fluctuation, Rotating Black Hole.

1. Introduction

In this paper, we will consider a special type of spiraling motion of a test particle down the black hole. We have found an expression for the convergence of null rays earlier. Here, we will find the expression for principal null rays. We take the metric a local from of reference, co-moving with the rotating black hole has been defined by Christodoulou and Ruffian as follows:

The angular velocity of the local frame with respect to an observer at rest at infinity is given by -

$$\omega = -\frac{g_{03}}{g_{33^2}} = \frac{F}{C}$$

The angular velocity of dragging of the inertial frame on the surface of the black hole tends to a common value in the limit, in which the collapsing object tends to its final

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stationary configuration. The 4-velocity of the observer co-moving with the system of reference are given by -

$$U^1 = 0,$$

$$U^2 = 0,$$

$$u^3 = d\phi/ds = \frac{-(g_{43}/g_{33})}{\frac{g_{43}^2}{g_{33}} - g_{44}} = \frac{F}{E\sqrt{C}}$$

$$u^4 = -(g^2_{43}/g_{33} - g_{44})^{-1/2} = \sqrt{C} / E,$$

where $E^2 = F^2 + CD = \Delta \, Sin^2\theta,$

A family of null geodesics, whose null tangent vectors are given by $l^i = dx^i/dv$ is considered, v being some parameter along the geodesics. We choose l^i as

$$l^i \left(\frac{1}{\sqrt{A}}, 0, \frac{F}{E\sqrt{C}}, \frac{\sqrt{C}}{E} \right)$$

At each point, a pair of unit space-like vectors a^i and b^i are chosen which are orthogonal to each other and to l^i a^i and b^i are specified as follows:

$$a^i \left(\frac{1}{\sqrt{A}}, \frac{1}{\sqrt{B}}, \frac{F}{E\sqrt{C}}, \frac{\sqrt{C}}{E} \right)$$

$$b^i \left(\frac{1}{\sqrt{A}}, 0, \frac{E+F}{E\sqrt{C}}, \frac{\sqrt{C}}{E} \right)$$

Then the complex conjugate vectors mi and mi are defined as follows:

$$\sqrt{2}m^i = a^i + ib^i$$

$$\sqrt{2}m^i = a^i - ib^i$$

The null vector n^i is specified as

$$n^i \left[\frac{1}{1\sqrt{A}}, \frac{1}{\sqrt{B}}, \frac{1}{C}, \left(\frac{3F}{2E} + 1 \right) \frac{3\sqrt{C}}{2E}, \right]$$

Then the convergence of a null rays are given by the real part of la, bm^a. m^{-b}. Calculating the real part of ☐ for the null tetr

$$\rho_{real} = -\frac{1}{4\sqrt{B}} \left(\frac{A_2}{A} + \frac{C_2}{C} - \frac{2E_2}{E} \right) + \frac{1}{4\sqrt{A}} \left(\frac{B_2}{B} + \frac{C_1}{C} \right)$$

$$= \frac{\sqrt{\Delta}}{2p} \left[\frac{2r(r^2+a^2-(\mathrm{r-m})a^2\,\mathrm{Sin}^2\theta + \sqrt{\Delta}a^2\mathrm{Sin}^2\theta\mathrm{Cos}\,\theta}{r^2+a^2)^2 - \Delta a^2\mathrm{Sin}^2\theta} \right]$$

with suffixes 1 and 2 denoting ordinary differentiation with respect to r and θ respectively.

Thus, we get an expression for the convergence of null emanating radically with respect to the emitter commoving with the rotating black hole.

2. Tangent Vector of the Null Geodesics

The problem is further simplified by considering the source and the photon to move with equatorial plane θ r π/2 of the black hole. Hence, we proceed to specify the orbit of the emitter and photon orbit in this plane.

The tangent vector of the null geodesics along which the photon propagate is taken as follows:

$$b^i \left(\frac{p}{\sqrt{A}} , o, \frac{pF}{E\sqrt{c}} , \frac{p\sqrt{C}}{E} \right)$$

Where p is a function of r only. It can easily be verified that the space-like vectors a^i and bi as defined are orthogonal to k^i.

If ρ' be the new value of ρ_{real} we have,

$$\rho' = -\frac{1}{\sqrt{A}} , \frac{\delta p}{\delta r} + \frac{p\sqrt{\Delta}}{2r} \left[\frac{2r(r^2+a^2-(\mathrm{r-m})a^2}{r^2+a^2)^2 - \Delta a^2} \right]$$

where θ = π/2

Now, p is subjected to the following condition,

$$- \ -\frac{\sqrt{\Delta}}{r}, \frac{\delta p}{\delta r} + \quad \frac{p\sqrt{\Delta}}{2r} \left[\frac{2r(r^2+a^2-(\mathrm{r-m})a^2}{r^2+a^2)^2-\Delta a^2} \right]$$

$$= \left[\sqrt{\Delta}/r^3\right]\left[(r^2 + a^2)^2 - \Delta a^2\right]^{1/4}$$

Solving the differential equation, we get

$$p = \left[\frac{r^2+a^2)^2-\Delta a^2}{\mathrm{r}}\right]^{1/4}$$

For the orbit of the emitting particle, we choose a time-like geodesic whose tangent vectors are given by,

$$U^i = \left(\frac{\in\sqrt{p^2-1}}{\sqrt{A}}, \ 0, \ \frac{pF}{E\sqrt{c}}, \ \frac{p\sqrt{C}}{E} \right)$$

where $\in = \pm 1$.

The angular velocity of the particle in the orbit with respect to observer at infinity is given by

$$d\phi/dt = F/C = \omega$$

Hence the angular velocity of the particle is the same as that of the local frame co-moving with the black hole and defined as above. Also from above, it can be said that the angular velocity of the photon being propagated in the orbit given by is ω. Thus, for the local observer defined as above. The particle is moving radically and the photons emitted by it are being propagated readily. For thru observer at rest at infinity, the radiating particles spiraling down ($\in = -1$) the black hole in the equatorial plane along the time like geodesic and the null rays which have past end points on this particle are tangents to the null geodesics.

3. Maximally Rotating Kerr Black Hole

It is assumed that the emitting particle has negligible effects on the background metric . Now we proceed to find out the frequency shift of the emitted radiation from the particle moving in the equatorial plane and whose trajectory is given by above equations..

The ratio of emitted to the observed frequency is given by Trautman and others as

$$1 + Z = \frac{k_1 u^i)_e}{k_1 u^i)_o}$$

The observer at infinity is chosen as static with 4-velocity as:

$$u^i_0 = \delta^i_0$$

We have

$$1 + Z = p \quad \sqrt{p^2-1} + p^2$$

For maximally rotating Kerr black hole (a = m), we have by retaining terms up to second order of ‘m/r’,

$$p = 1 + \frac{1}{4} \qquad \frac{m^2}{r^2}$$

$$1 + Z = 1 + \frac{1}{\sqrt{2}} \qquad \frac{m}{r} \quad + \qquad \frac{1}{2} \qquad m^2 r^2$$

For different values of r and the corresponding values for (1 + Z) we have, for the escaping photons, the following table.

Table - 1

r	m	2m	3m	4m	5m	6m	7m
1+Z	4.5	3.7	2.9	2.03	1.10	0.70	0.60

A look on the above table gives the interesting feature that there is high spread in (1+Z) among the photons emitted at small values of ‘r’ and that the spread in (1+Z) decreases slowly as the distance increases from r = 2m and onwards. Below the Schwarzschild radius the frequency shift changes abnormally. it is evident that (1+Z) never assumes the value less than 1. Hence “Blue shifted radiation” is ruled out in this model of accreting particles. The intensity and frequency of radiation is affected more in the lower region, m <– r <– 2m of the black hole.

We proceed to evaluate the co-ordinate orbital period of the emitter as measured in the observer’s clock.

By assuming a = m and retaining terms upto the order of (m/r)2, gives for $\in$ = -1, (ingoing test particle),

$$\frac{dr}{dr} = - \quad \frac{1}{\sqrt{2}} \quad \frac{m}{r} \left(1 - \frac{2m}{r} \right)$$

$$\frac{d\varphi}{dt} = -2\sqrt{2} \ \frac{m}{r^2} \left(1 - \frac{2m}{r} \right)$$

From these two expressions, the period p can be written as

$$P = \frac{\pi \int_{r+h}^{r} \frac{r^2 \mathrm{dr}}{m \ (r-m}}{\int_{r+h}^{r} \left(\frac{m}{r^2} + \frac{2m^2}{r^3} \right) \mathrm{dr}}$$

$$= \frac{\pi r^2}{m^2} \left(1 + \frac{4r^2}{\mathrm{r}^2} \right)$$

Evaluating P for different values of r we get the following table:

Table - 2

r	P
m	5πm = (0.08msec) (m/MΘ)
2m	16πm = (0.20msec) (m/MΘ)
3m	39πm = (0.60msec) (m/MΘ)
4m	80πm = (1.20msec) (m/MΘ)
5m	145πm = (2.20msec) (m/MΘ)
6m	240πm = (3.70msec) (m/MΘ)
7m	371πm = (5.70msec) (m/MΘ)

Doppler shift and focusing by the black hole cause the frequency and intensity of radiation to fluctuate with the orbital period. The fluctuation period of intensity and frequency of radiation from the particle falling into the black hole is the orbital period of the particle. The minimum fluctuation period is the orbital period at the stable circular orbit. It has been observed by Swayne (48) that if hot spots occur on the surface of a disc of material accreting into the black hole, the X-rays received from the disc should exhibit a characteristic quasi periodic variability.

He has found that the minimum fluctuation period for the radiation from the disc should be eight time as short for rotating as for non-rotating black hole of the same mass. The minimum fluctuation period for the two cases have been given as,

$$P_{min} = 12\pi\sqrt{6} \quad m = (0.50 \text{ msec})(m/M\Theta)$$
for non-rotating hole

$$P_{min} = 4\pi m \quad = (0.06 \text{ msec})(m/M\Theta)$$
for maximally rotating hole

The limit set by Swayne for P_{min} for a maximally rotation black hole is

$$[(0.06\text{msec})(m/M\Theta) \lesssim P_{min} \lesssim (0.50\text{msec})(m/M\Theta)]$$

This also served as a test for rotation of black hole. It is evident from the above, Table - 9, that P_{min} in our case confirms to the limit set by Swayaev. From the table 9, it appears that in the beginning as the rate of decrease in the fluctuation period decreases steeply the rate of decrease in the fluctuation period is not so steep in the lower region.

It would also be interesting to find out which of the photons emitted at generic point on the orbit escape to infinity and reach the observer with help of geometrical optics. The photons after being emitted from the generic point on the orbit reach the observer after making some revolutions round the black hole. The algebraic sum of ω-revolving and the counter revolving one,(the former contributes positively) by the photon after it is emitted is given by,

$$n = -\frac{1}{2\pi} \int_{re}^{\infty} \frac{k^2}{k^1} \, dr$$

$$= \frac{1}{\pi} \cdot \frac{m^2}{2re^2}$$

The angular position of the emitter on the orbit is then given by,

$$\phi = \frac{360^0}{\pi} \quad . \quad \frac{m^2}{2re^1} \quad . \quad \text{in degrees}$$

Evaluating ϕ for various values of re we get the following table.

Table – 3

r_e	m	2m	3m	4m	5m	6m	7m
ϕ	57.2°	19.3°	6.3°	3.6°	2.3°	1.6°	1.2°

From the table-10, it appears that the angular distance of generic point on the orbit from the line ϕ = 0 increases more and more as the particle is approaching nearer the event horizon. The greatest angular distance is about 57.2°, when the particle is on the last orbit. This is natural as the focusing effect of gravity on the photon ray will be more and more pronounced as the particle is moving nearer to the horizon.

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